CE 205: Finite Element Method: Homework II

Instructor: Dr. Narayan Sundaram*

January 30, 2024

(In this version, a typo has been corrected in the distributed load in problem I.)
You must submit your source code / scripts as part of your submission for all coding problems. Show all work clearly. Plots and diagrams must be labeled legibly and completely. This homework has THREE pages.

1. Consider an elastic rod of length L with uniform sectional properties EA. It is fixed at one end ($x = 0$) and subjected to a point load $P > 0$ at $x = L/2$. It is also subjected to a distributed axial load per unit length $q(x) = q_0 \sin(2\pi x/L)$ along its entire length, where $5q_0 L = 4P$. Solve the rod governing differential equation to find the displacement field $u(x)$ and stresses $\sigma(x)$ in the rod.

2. Next, using 2- and 4- finite elements along the length of the rod in Problem (I):
   (a) Write down the global and reduced stiffness matrices $[K]$, $[K^*]$ and the global d.o.f vector $\{u\}$
   (b) Find the mid-length displacement $u(x = L/2)$ for 2- and 4- element analyses
   (c) For both 2- and 4- element cases, find and sketch
   (i) The global displacement $u(x)$
   (ii) Element stresses as a function of x

3. *Coding exercise* Write a program in MATLAB, C, or any other language of your choice to automate the FE solution to problem (I) for *any* (user-specified) number of elements $N \geq 2$ which has at least one node at $x = L/2$. For the FE coding exercise, you can take $L = 2$ m, $E = 70$ GPa, $A = 10$ mmsq, P=1000 N, $q_0 = 800 N/m$.

   (a) Make a table of the consistent nodal forces corresponding only to the distributed load $q(x)$ for N=2, 4, and 8 nodes. The consistent nodal force must be determined by your FE code.

*Department of Civil Engineering, Indian Institute of Science

(b) Use your code to find and plot the mid-length displacement $u^{FE}(x = L/2)$ against the number of elements for N=2, 4, 8, and 16 elements. What do you observe?

(c) Make a plot which compares the FE solutions at various N, $u^{FE}(x)[N]$ to the analytical formula for $u(x)$ from problem (I). Your plot should have five curves (four FE, one analytical solution) on the same axes.

(d) Make a table of $u^{FE}/u\big|_{(x=L/2)} - 1$ for different values of N.

(e) Repeat the above comparison exercise for the element stresses in the rod. What differences do you notice between displacements and stresses as N is increased?

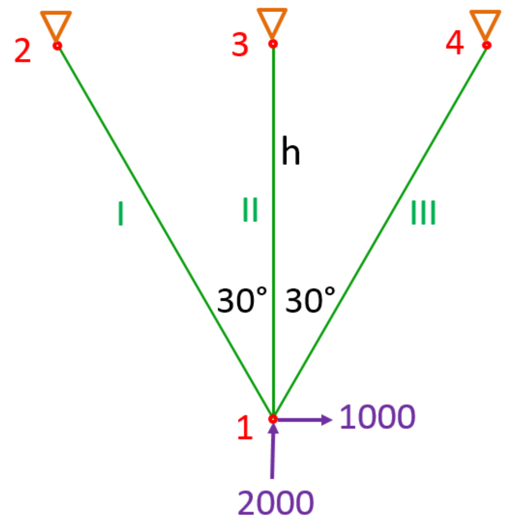

Figure 1: Three-truss problem

4. Consider the first plane three-truss problem in Fig. 1, which was solved in class. All elements have the same cross-sectional EA.

(a) Re-derive the reduced stiffness matrix $[K^*]$ and solve for the displacements (u_1, v_1) at node 1.

(b) Next, find the entries of the $[K_{BA}]$ system sub-matrix and use it to find the nodal forces at nodes 2, 3, and 4.

(c) Find the stresses in the three elements.

5. Consider the equilateral plane truss problem discussed in class, and shown in Fig. 2. The members are all of length L and the cross-sectional properties EA.

   (a) Write down the reduced stiffness matrix $[K^*]$ and solve for the unknown degrees-of-freedom u_2, u_3, v_3.

   (b) Find the reactions at nodes 1 and 2.

   (c) Find the stresses in the elements.

   (d) Discuss what happens when the pinned connection at node 1 is replaced by rollers.

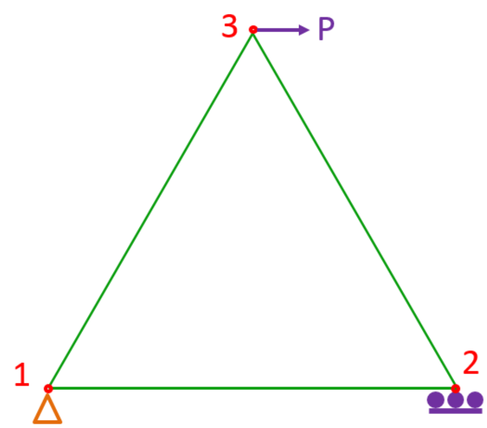

Figure 2: Equilateral truss problem